Compositions of Isometries

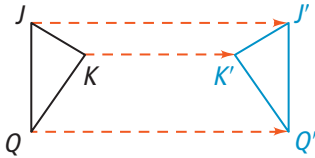## Vocabulary

Review

Use the transformation below to answer questions 1-3.

1. Fill in the blanks.

   The *image* of ∠J is ☐ and the *image* of ∠K is ☐ .

2. Circle the pair of *corresponding sides*.

   $\overline{JQ}$ and $\overline{J'Q'}$ $\overline{KQ}$ and $\overline{J'Q'}$ $\overline{JK}$ and $\overline{J'Q'}$ $\overline{JQ}$ and JQ'

3. Circle the word that completes the sentence.

   The *transformation* $\triangle JKQ \rightarrow \triangle J'K'Q'$ is a _____.

   translation   reflection   rotation   dilation

Vocabulary Builder

Isometry (noun) ī sä mə trē

Related Words: transformation, translation, reflection, rotation, glide reflection

Definition: An **isometry** is a transformation that preserves distance, or length.

Example: Translations, reflections, rotations, and glide reflections are **isometries**.

Use Your Vocabulary

4. Underline the correct word to complete each sentence.

   In a transformation, the original figure is the image/preimage .

   The resulting figure is the image/preimage .

5. Circle the type of transformation that maps each (x, y) to $(x - 8, y + 2)$.

   translation   reflection   rotation   glide reflection   dilation
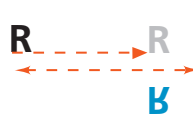
There are only four isometries.

Translation   **Rotation**   **Reflection**   **Glide Reflection**

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take note

Theorem 9-1

The composition of two or more isometries is an isometry.

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take note

Theorem 9-2   Reflections Across Parallel Lines

A Composition of reflections across two parallel lines is a translation.

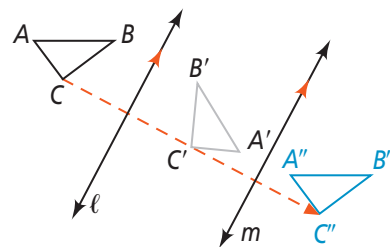You can write this composition as
$R_m \circ R_l(\triangle ABC) = \triangle A''B''C''$
or $R_m(R_l(\triangle ABC)) = \triangle A''B''C''$.
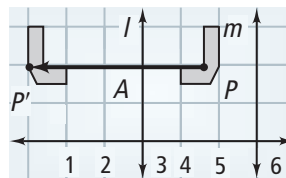
The Composition has the following properties.

- $\overline{AA''}$, $\overline{BB''}$, and $\overline{CC''}$ are all perpendicular to lines l and m.
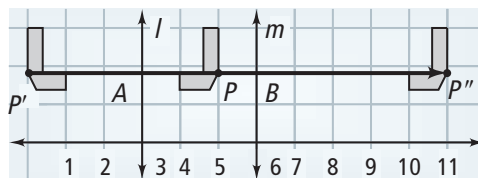
- $AA'' = BB'' = CC'' = 2PQ$

Problem 1 **Composing Reflections Across Parallel Lines**

Got It? Draw parallel lines l and m. Draw J between l and m. What is the image of $(R_m \circ R_l)(J)$? What is the distance of the resulting translation?

6. Reflect J across line l. $PA = AP'$, so $PP' = 2 \boxed{}$.

7. Reflect the image across line m. $P'B = BP''$, so $P'P'' = 2 \boxed{}$.

8. Circle the correct answer.
P moved a total distance $PP'' = P'P'' - PP' = 2BP' - 2AP' = \boxed{?}$

| 0.5AB | AB | 1.5AB | 2AB | 2.5AB |

Lesson 9-4

Theorem 9-3 Reflections Across Intersecting Lines

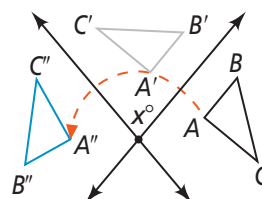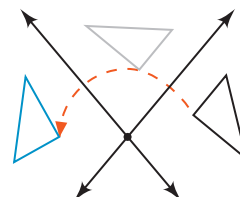A Composition of reflections across two intersecting lines is a rotation.

You can write this composition as $(R_l \circ R_m)(\triangle ABC) = \triangle A''B''C''$

$R_l(R_m \triangle ABC) = \triangle A''B''C''$.

The composition has the following properties.

- The figure is rotated about the point where the two lines intersect. In this case, point Q.

9. Underline the correct word to complete the sentence. $\triangle ABC$ is rotated

clockwise/counterclockwise around the point of intersection.

Problem 3 **Finding a Glide Reflection Image**

Got It? $\triangle TEX$ is shown in the graph to the right. What is the image of $\triangle TEX$ for the glide reflection $(R_{y=-2} \circ T_{<1,0>})(\triangle TEX)$?

Use the information you are given and choose from the following words to fill in the blanks and complete each statement.

reflection   *translate*   *reflect*   *image*

Know

Vertices of $\triangle TEX$:

$T(\boxed{},\boxed{})$, $E(\boxed{},\boxed{})$,

$X(\boxed{},\boxed{})$
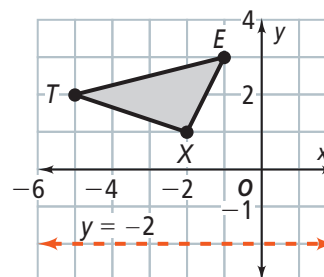
Translation rule: $T_{<\square,\square>}$ $(\triangle TEX)$

Reflection line equation:

_____

Need

The _____ of $\triangle TEX$ for the glide reflection

Plan

First use the translation rule to _____ $\triangle TEX$. Then _____ the translation image of each vertex across the line of _____.

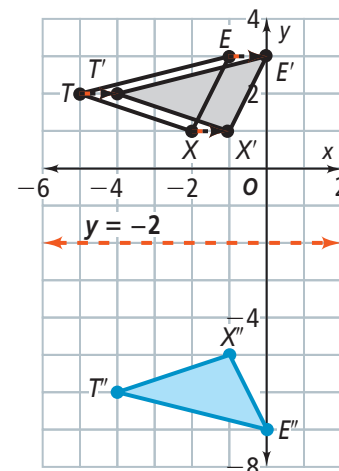**10.** Underline the correct word to complete the sentence.

Use the translation rule $T_{<1,0>}(\triangle TEX)$ to move $\triangle TEX$

right / left / up / down 1 unit.

11. Fill in the blanks to make correct statements.

Reflect the image of $\triangle T'E'X'$ across the line $\boxed{}$. The vertices of $\triangle A''B''C''$ are

$T''(\boxed{},\boxed{})$, $E''(\boxed{},\boxed{})$, $X''(\boxed{},\boxed{})$.
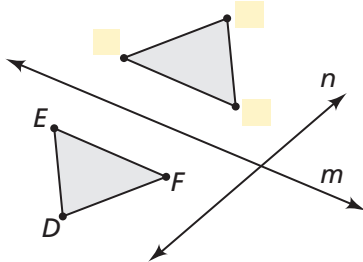
Lesson Check • Do you UNDERSTAND?

Error Analysis You reflect △DEF first across line *m* and then across line *n*. Your friend says you can get the same result by reflecting △DEF first across line *n* and then across line *m*. Explain your friend's error.
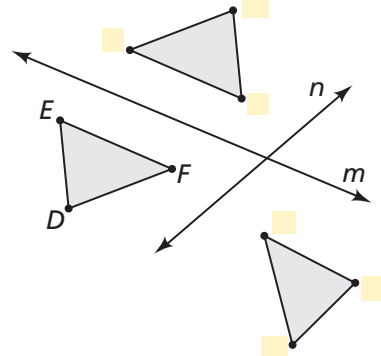
Fill in the blanks in each statement and diagram.
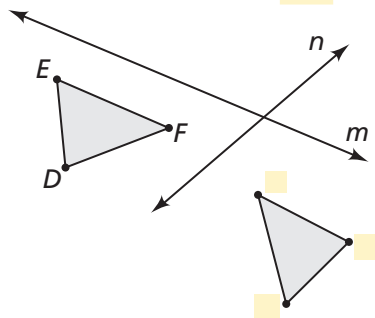
You:

12. Reflect △DEF over line ☐.

13. Reflect the image of △DEF over line ☐.

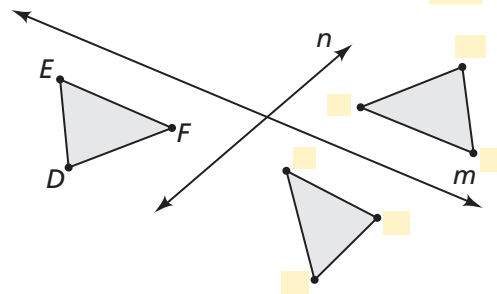**Your friend:**

14. Reflect △DEF over line ☐.

15. Reflect the image of △DEF over line ☐.

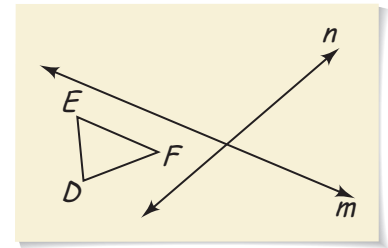**16.** Explain your friend's error on the lines below.
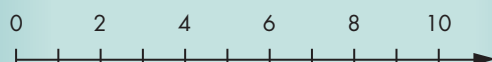
_____

_____

_____

Math Success

Check off the vocabulary words that you understand.

☐ glide reflection   ☐ isometry

Rate how well you can *use compositions of isometries*.

| Need to review | 0 | 2 | 4 | 6 | 8 | 10 | Now I get it! |