

12.4 - Average rates of Change. p. 2

Determine a) the Net Change $f(b) - f(a)$
 b) the Average Rate of Change $\frac{f(b) - f(a)}{b - a}$

ex. 1

$$f(x) = 3x - 2 ; x = 2, x = 3$$

a) $f(2) = 6 - 2 = 4$

$f(3) = 9 - 2 = 7$

$7 - 4 = 3$

b) $f(b) - f(a) = \frac{3}{3-2} = \frac{3}{1} = 3$

ex. 2

$$f(x) = 2x^2 - x ; x_1 = 3, x_2 = 6$$

a) $f(3) = 2(3)^2 - (3)$

$= 2(9) - 3$
 $\rightarrow = 18 - 3 = 15$

$f(6) = 2(6)^2 - (6)$

$= 2(36) - 6 =$
 $\rightarrow = 72 - 6 = 66$

66 - 15
 = 51
 net change

b) $\frac{51}{6-3} = \frac{51}{3} = 17$

Avg. Rate of Change

ex. 3

$$f(x) = x^3 - 4x^2 ; x = 0, x = 10$$

a) $f(0) = 0^3 - 4(0)^2$
 $= 0$

$f(10) = (10)^3 - 4(10)^2$

$= 1000 - 400$

$= 600$

net Change

$600 - 0 = 600$

b) $\frac{600}{10-0} = \frac{600}{10}$

$= 60$

Avg. rate of Change

ex. 4

$$f(x) = 1 - 3x^2 ; \quad x = 2, \quad x = 2 + h$$

$$f(2) = 1 - 3(2)^2$$

$$= 1 - 3(4)$$

$$= 1 - 12 = -11 \quad \text{FAIL}$$

$$f(2+h) = 1 - 3(2+h)^2$$

$$= 1 - 3(4 + 4h + h^2)$$

$$= -11 - 12h - 3h^2$$

Net change

$$\frac{f(b) - f(a)}{b - a} = \frac{-11 - 12h - 3h^2 - (-11)}{(2+h) - (2)}$$

$$= \frac{-12h - 3h^2}{h}$$

$$= \frac{-12h - 3h^2}{h} = -12 - 3h$$

Running Speed A man is running around a circular track that is 200 m in circumference. An observer uses a stopwatch to record the runner's time at the end of each lap, obtaining the data in the following table.

- (a) What was the man's average speed (rate) between 68 s and 152 s?
- (b) What was the man's average speed between 263 s and 412 s?
- * Calculate the man's speed for each lap. Is he slowing down, speeding up, or neither?

Time (s)	Distance (m)
32	200
68	400
108	600
152	800
203	1000
263	1200
335	1400
412	1600

$$x_1 = 68$$

$$x_2 = 152$$

$$y_1 = 400$$

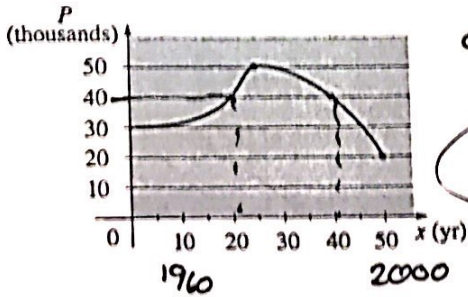
$$y_2 = 800$$

$$\frac{800_m - 400_m}{152_s - 68_s} = \frac{400}{84} \approx 4.8 \frac{m}{s}$$

units

Population Growth and Decline The graph shows the population P in a small industrial city from 1950 to 2000. The variable x represents the number of years since 1950.

- (a) What was the average rate of change of P between $x = 20$ and $x = 40$?
- (b) Interpret the value of the average rate of change that you found in part (a).



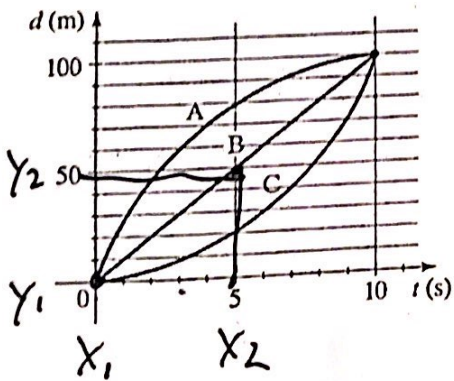
a) $x_1 = 20$ $y_1 = 40$
 $x_2 = 40$ $y_2 = 40$

$$\frac{40 - 40}{40 - 20} = \frac{0}{20} = 0$$

Three-Way Tie A downhill skiing race ends in a three-way tie for first place. The graph shows distance as a function of time for each of the three winners, A, B, and C.

- (a) Find the average speed for each skier
- (b) Describe the differences between the ways in which the three participants skied the race.

Ben



a) $m_B = \frac{50 - 0}{5 - 0} = 10 \frac{m}{s}$